There are many interactions between noncommutative algebra and representation theory on the one hand and classical algebraic geometry on the other, with important applications in both directions. The aim of this book is to provide a comprehensive introduction to some of the most significant topics in this area, including noncommutative projective algebraic geometry, deformation theory, symplectic reflection algebras, and noncommutative resolutions of singularities.

The book is based on lecture courses in Noncommutative Algebraic Geometry given by the authors at a Summer Graduate School at MSRI in 2012 and, as such, is suitable for advanced graduate students and early postdocs. In keeping with the lectures on which the book is based, a large number of exercises are provided, for which partial solutions are included.

### Mathematical Sciences Research Institute Publications

**64** 

Noncommutative Algebraic Geometry

#### Mathematical Sciences Research Institute Publications

- Freed/Uhlenbeck: Instantons and Four-Manifolds, second edition 1
- 2 Chern (ed.): Seminar on Nonlinear Partial Differential Equations
- Lepowsky/Mandelstam/Singer (eds.): Vertex Operators in Mathematics and Physics 3
- 4 Kac (ed.): Infinite Dimensional Groups with Applications
- Blackadar: K-Theory for Operator Algebras, second edition 5
- Moore (ed.): Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics 6
- Chorin/Majda (eds.): Wave Motion: Theory, Modelling, and Computation
- 8 Gersten (ed.): Essays in Group Theory
- Moore/Schochet: Global Analysis on Foliated Spaces, second edition
- 10-11 Drasin/Earle/Gehring/Kra/Marden (eds.): Holomorphic Functions and Moduli
- 12-13 Ni/Peletier/Serrin (eds.): Nonlinear Diffusion Equations and Their Equilibrium States
  - 14 Goodman/de la Harpe/Jones: Coxeter Graphs and Towers of Algebras
  - 15 Hochster/Huneke/Sally (eds.): Commutative Algebra
  - 16 Ihara/Ribet/Serre (eds.): Galois Groups over Q
  - 17 Concus/Finn/Hoffman (eds.): Geometric Analysis and Computer Graphics
  - Bryant/Chern/Gardner/Goldschmidt/Griffiths: Exterior Differential Systems 18
  - 19 Alperin (ed.): Arboreal Group Theory
  - Dazord/Weinstein (eds.): Symplectic Geometry, Groupoids, and Integrable Systems 20
  - 21 Moschovakis (ed.): Logic from Computer Science
  - Ratiu (ed.): The Geometry of Hamiltonian Systems 22
  - Baumslag/Miller (eds.): Algorithms and Classification in Combinatorial Group Theory 23
  - 24 Montgomery/Small (eds.): Noncommutative Rings
  - 25 Akbulut/King: Topology of Real Algebraic Sets
  - 26 Judah/Just/Woodin (eds.): Set Theory of the Continuum
  - Carlsson/Cohen/Hsiang/Jones (eds.): Algebraic Topology and Its Applications 27
  - 28 Clemens/Kollár (eds.): Current Topics in Complex Algebraic Geometry
  - 29 Nowakowski (ed.): Games of No Chance
  - 30 Grove/Petersen (eds.): Comparison Geometry
  - 31 Levy (ed.): Flavors of Geometry
  - Cecil/Chern (eds.): Tight and Taut Submanifolds 32
  - 33 Axler/McCarthy/Sarason (eds.): Holomorphic Spaces
  - Ball/Milman (eds.): Convex Geometric Analysis 34
  - 35 Levy (ed.): The Eightfold Way
  - Gavosto/Krantz/McCallum (eds.): Contemporary Issues in Mathematics Education 36
  - 37 Schneider/Siu (eds.): Several Complex Variables
  - 38 Billera/Björner/Green/Simion/Stanley (eds.): New Perspectives in Geometric Combinatorics
  - 39 Haskell/Pillay/Steinhorn (eds.): Model Theory, Algebra, and Geometry
  - Bleher/Its (eds.): Random Matrix Models and Their Applications
  - Schneps (ed.): Galois Groups and Fundamental Groups 41
  - 42 Nowakowski (ed.): More Games of No Chance
  - 43 Montgomery/Schneider (eds.): New Directions in Hopf Algebras
  - 44 Buhler/Stevenhagen (eds.): Algorithmic Number Theory: Lattices, Number Fields, Curves and Cryptography
  - Jensen/Ledet/Yui: Generic Polynomials: Constructive Aspects of the Inverse Galois Problem 45
  - 46 Rockmore/Healy (eds.): Modern Signal Processing
  - 47 Uhlmann (ed.): Inside Out: Inverse Problems and Applications
  - 48 Gross/Kotiuga: Electromagnetic Theory and Computation: A Topological Approach
  - Darmon/Zhang (eds.): Heegner Points and Rankin L-Series 49
  - 50 Bao/Bryant/Chern/Shen (eds.): A Sampler of Riemann-Finsler Geometry
  - 51 Avramov/Green/Huneke/Smith/Sturmfels (eds.): Trends in Commutative Algebra
  - Goodman/Pach/Welzl (eds.): Combinatorial and Computational Geometry 52
  - 53 Schoenfeld (ed.): Assessing Mathematical Proficiency
  - 54 Hasselblatt (ed.): Dynamics, Ergodic Theory, and Geometry
  - 55 Pinsky/Birnir (eds.): Probability, Geometry and Integrable Systems
  - 56 Albert/Nowakowski (eds.): Games of No Chance 3
  - 57 Kirsten/Williams (eds.): A Window into Zeta and Modular Physics
  - 58 Friedman/Hunsicker/Libgober/Maxim (eds.): Topology of Stratified Spaces
  - 59 Caporaso/M<sup>C</sup>Kernan/Mustață/Popa (eds.): Current Developments in Algebraic Geometry

  - 60 Uhlmann (ed.): Inverse Problems and Applications: Inside Out II 61
  - Breuillard/Oh (eds.): Thin Groups and Superstrong Approximation 62 Eguchi/Eliashberg/Maeda (eds.): Symplectic, Poisson, and Noncommutative Geometry
  - 63 Nowakowski (ed.): Games of No Chance 4

  - Bellamy/Rogalski/Schedler/Stafford/Wemyss: Noncommutative Algebraic Geometry 64
- Deift/Forrester (eds.): Random Matrix Theory, Interacting Particle Systems, and Integrable Systems 65 67–68 Eisenbud/Iyengar/Singh/Stafford/Van den Bergh (eds.): Commutative Algebra and Noncommutative Algebraic Geometry

## Noncommutative Algebraic Geometry

### Gwyn Bellamy

University of Glasgow

# Daniel Rogalski

University of California, San Diego

### Travis Schedler

Imperial College London

### J. Toby Stafford University of Manchester

Michael Wemyss



Gwyn Bellamy gwyn.bellamy@glasgow.ac.uk Daniel Rogalski drogalsk@math.edu Travis Schedler schedler@math.utexas.edu

J. Toby Stafford toby.stafford@manchester.ac.uk Michael Wemyss m.wemyss@ed.ac.uk

Silvio Levy (*Series Editor*) Mathematical Sciences Research Institute levy@msri.org

The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation for the publication of this series.

#### **CAMBRIDGE** UNIVERSITY PRESS

32 Avenue of the Americas, New York, NY 10013-2473, USA

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107129542

© Mathematical Sciences Research Institute 2016

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2016

A catalog record for this publication is available from the British Library.

ISBN 978-1-107-12954-2 Hardback 978-1-107-57003-0

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party Internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.