This book provides an introduction, with applications, to three interconnected mathematical topics:

- zeta functions in their rich variety: those of Riemann, Hurwitz, Barnes, Epstein, Selberg, and Ruelle, plus graph zeta functions;
- modular forms: Eisenstein series, Hecke and Dirichlet L-functions, Ramanujan's tau function, and cusp forms;
- vertex operator algebras: correlation functions, quasimodular forms, modular invariance, rationality, and some current research topics including higher-genus conformal field theory.

Applications of the material to physics are presented, including Kaluza–Klein extra-dimensional gravity, bosonic string calculations, a Cardy formula for black hole entropy, Patterson–Selberg zeta function expressions of one-loop quantum field and gravity partition functions, Casimir energy calculations, atomic Schrödinger operators, Bose–Einstein condensation, heat kernel asymptotics, random matrices, quantum chaos, elliptic and theta function solutions of Einstein's equations, a soliton–black hole connection in two-dimensional gravity, and conformal field theory.

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A Window into Zeta and Modular Physics

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