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Chapter 2
Aims of Mathematics Education

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Education is our means to instruct our youth in the values and accomplishments of our civilization and to prepare them for adult life. For centuries, arguments have been made about what an education means and how to distinguish an educated person from an uneducated one. Two views have contended for our allegiance since the time of the ancient Greeks [Marrou 1956]. One perspective is the rational and humane vision of the Sophists and later the philosopher-teacher Isocrates, for whom the test of an education was its ability to prepare a citizen to engage in public affairs. The other view is that of Plato and Socrates, who taught that education must guide the student toward an uncovering of the Truth and Beauty that underlie our human experience, the universal themes and natural laws that a well schooled mind can discern beneath the surface confusion of life and the awakening of the spirit within, that allow us to care intensely about life and learning.

We cannot clear up some of the controversies about mathematics education and how to assess learning until we deal with two underlying issues. The first is the mindset that underlies our approach to assessment. The other is to articulate and then discuss our often unspoken assumptions about what it means to be well educated.

First, let us consider what drives our current approaches to assessment. In a recent workshop on assessment [NRC 2003], the point was made that the public accountability movement is driving assessment toward increasingly large-scale tests of what students know. These tests "do not easily conform to curricula devised to match state and national standards" [NRC 2003, p. ix]. A basic problem is that testing has been shaped by psychometric questions (How can we measure this?) and used increasingly for political purposes rather than educational questions that can support learning (Is this worth measuring? What do students really need to know and can we measure that knowledge?). We must bridge the gap between what the large-scale tests measure and how the test results are interpreted and used, on the one hand, and what students and teachers are trying to accomplish in the classroom, on the other. To do this, we can profit by studying the NRC workshop report on assessment. It recommends that large-scale assessments and classroom assessments (a) share a common model of student learning, (b) focus on what is most highly valued rather than what is easy to measure, (c) signal to teachers and students what is important for them to teach and learn. The report goes on to offer some helpful technical and design elements that can increase the usefulness of both levels of tests.

If we are to assess what is most highly valued, we then must address the second underlying problem, namely, what *do* we value and what do we seek as the goal of education? We cannot talk about assessment until we are clear about our underlying philosophy of education and our goals for all of our young people. As long as we continue to approach the role of mathematics in the curriculum from different perspectives, we will have difficulty agreeing on what students should know and how they should learn. While we seek clarity of purpose, we need to keep in mind that our discussions must have genuine consequences for all students, including those that we do not serve well today. Robert Moses [2001] has made the case that children who are not quantitatively literate may be doomed to second-class economic status in our increasingly technological society. We have compelling evidence that “poor children and children of color are consistently shortchanged when it comes to mathematics” [Schoenfeld 2002]. Schoenfeld argues that we can serve all children well if we attend to four critical conditions in our schools.

- A high quality curriculum.
- A stable, knowledgeable and professional teaching community.
- High quality assessment that is aligned with curricular goals.
- Mechanisms for the continued development of curricula, assessment and professional development of our teachers and administrators.

To put all of these conditions in place, however, we need to develop a consensus about what it means to be mathematically literate. How shall we define “basic skills” and “conceptual understanding and problem solving,” the relationship of these things to each other, and the appropriate balance of the two in our curriculum?

In *The Great Curriculum Debate*, Tom Loveless traces our current dilemmas back to John Dewey, who, in 1902 described two “sects.” One subdivides every subject into studies, then lessons, and finally specific facts and formulae; the other focuses on the development of the child and active learning. Loveless describes the first sect as the educational-traditionalist mode or *rational* mode, a teacher-centered model that seeks classical explicit goals, expects discipline and order in the classroom where the class is led by the teacher and students are assessed by regular testing.

According to Loveless [2001, p. 2], "Traditionalists are skeptical that children naturally discover knowledge or will come to know much at all if left to their own devices." They are "confident that evidence, analysis and rational thought are greater assets in the quest for knowledge and virtue than human intuition and emotions" [2001, p. 3].

Loveless characterizes the child-centered model as the educational progressive or *romantic* tradition that "reveres nature and natural learning and allows learning to unfold without standards, rules, hierarchies of skill, rote practice and memorization." Critics of the "traditional view" tend to describe it as "drill and kill." Critics of reform-oriented mathematics dismiss it as "fuzzy math" and point out errors in the mathematics itself, arguing that the curriculum does not make mathematical sense. It is clear that the problem we have in deciding how to assess mathematics is that *we do not agree on a philosophy of education* that can offer guidance about what should be taught and how, and most importantly, *for what reasons*.

A recent study by the Mathematics Learning Study Committee of the National Research Council draws on elements of both traditions, "the basics" as well as "conceptual understanding," and links them together into a larger vision of what it means to know and be able to use mathematics. Perhaps this more integrative model can move us toward a shared understanding about what mathematics must be taught, how and to what end. Only then can we really agree on how to go about assessing mathematics learning.

The NRC in its booklet *Helping Children Learn Mathematics* [Kilpatrick and Swafford 2002] summarizes the exploration of mathematics education that appeared in fuller form in *Adding It Up: Helping Children Learn Mathematics* (2001). The "interwoven and interdependent" components of mathematics proficiency advanced by the NRC Committee are:

- Understanding: Comprehending mathematical concepts, operations and relations and knowing what mathematical symbols, diagrams and procedures mean.
- Computing: Carrying out mathematical procedures, such as adding, subtracting, multiplying and dividing numbers flexibly, accurately, efficiently and appropriately.
- Applying: Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
- Reasoning: Using logic to explain and justify a solution to a problem or to extend from something known to something unknown.

- Engaging: Seeing mathematics as sensible, useful and doable — if you work at it — and being willing to do the work.

[Kilpatrick and Swafford 2002, p. 9]

This balanced approach is consistent with the work of Jerome Bruner, who argued that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" [Bruner 1977, p. 33].

Roger Geiger [1993] has pointed out that we academics tend to picture ourselves as "communities of scholars, free and ordered spaces, dedicated to the unfettered pursuit of teaching and learning." According to Geiger, in these intellectual spheres we produce increasingly specialized knowledge, not "wisdom, sagacity, or liberal learning." In recent years, there has been a great deal of exploration about how to link theory and utility in the scholarly pursuits of both faculty and students. The concepts of Donald Stokes are worth considering as a starting point for making peace across intellectual domains and clarifying our expectations about what we expect students to learn about mathematics. Although his approach was directed toward technology transfer, his ideas apply equally well to the design of the curriculum and its goals.

Stokes [1997] sought a "more realistic view of the relationship between basic research and technology policies" (p. 2) and hence between private interests (those of the researcher) and the public good (the advance of technology and its effects on society and the economy). To pave the way toward an effective blending of the imperative of knowledge for its own sake and knowledge that has consequences, Stokes developed the concept of intellectual spaces that he calls *Quadrants*. These are defined by the balance of theoretical interests and practical use pursued. Thomas Alva Edison's work fits nicely into the space framed by high interest in use and low interest in advancing understanding. He was "the applied investigator wholly uninterested in the deeper scientific implication of his discoveries." As Stokes puts it, "Edison gave five years to creating his utility empire, but no time at all to the basic physical phenomena underlying his emerging technology" [Stokes 1997, p. 24].

Niels Bohr represents the classic researcher engaged in a search for pure understanding as he explored the structure of the atom. For him, any possible practical use of his modeling was not even a consideration.

Occupying the quadrant where theory and use reinforce each other is Louis Pasteur, who had a strong commitment to understanding the underlying microbiological processes that he had discovered and, simultaneously, a motivation to use that knowledge to understand and control food spoilage, support the French wine industry and treat microbial-based disease.

It would be helpful if our discussions about mathematics education took place in Pasteur's Quadrant while recognizing that some of us are more comfortable in either Bohr's Quadrant or Edison's Quadrant. Some of our students will be drawn to deeper study of mathematics and become academic mathematicians. Others will want to develop a deeper understanding of mathematics in order to pursue careers in science, technology, or engineering. They will need to have a capacity for problem-solving and quantitative reasoning in their repertoire, but they will not advance our understanding of mathematics or pursue careers that have a rich mathematical base. Although we academics must serve all students well, they will ultimately use mathematics in very different ways. We must keep all of our students in mind and teach them authentically and honestly, being faithful to the discipline of mathematics and mindful of our students and how they are developing.

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