During the past few decades, the gradual merger of Discrete Geometry and the newer discipline of Computational Geometry has provided enormous impetus to mathematicians and computer scientists interested in geometric problems. This volume, which contains 32 papers on a broad range of topics of current interest in the field, is an outgrowth of that synergism. It includes surveys and research articles exploring geometric arrangements, polytopes, packing, covering, discrete convexity, geometric algorithms and their complexity, and the combinatorial complexity of geometric objects, particularly in low dimension. There are points of contact with many applied areas such as mathematical programming, visibility problems, kinetic data structures, and biochemistry, as well as with algebraic topology, geometric probability, real algebraic geometry, and combinatorics.

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#### Volumes 1–4 and 6–27 are published by Springer-Verlag

# Combinatorial and Computational Geometry

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The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation. This material is based upon work supported by NSF Grant 9810361.

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press 40 West 20th Street, New York, NY 10011-4211, USA

www.cambridge.org

Information on this title: www.cambridge.org/9780521848626

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First published 2005

Printed in the United States of America

A catalogue record for this book is available from the British Library.

Library of Congress Cataloging in Publication data

Combinatorial and computational geometry / edited by Jacob E. Goodman, János Pach, Emo Welzl.

p. cm. – (Mathematical Sciences Research Institute publications ; 52) Includes bibliographical references and index.

ISBN 0-521-84862-8 (hb)

Discrete geometry. 2. Combinatorial geometry. 3. Geometry–Data processing.
I. Goodman, Jacob E. II. Pach, János. III. Welzl, Emo. IV. Series.

QA640.7.D54 2005 516'.11-dc22

#### 2005042199

ISBN-13 978-0-521-84862-6 hardback ISBN-10 0-521-84862-8 hardback

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