Finsler geometry generalizes Riemannian geometry in the same sense that Banach spaces generalize Hilbert spaces. This book presents an expository account of seven important topics in Riemann–Finsler geometry, which have recently undergone significant development but have not had a detailed pedagogical treatment elsewhere. Each article will open the door to an active area of research and is suitable for a special topics course in graduate-level differential geometry.

Álvarez and Thompson discuss the theory of volumes for normed spaces and Finsler spaces and show how it unifies a wide range of geometric inequalities. Bellettini studies the evolution of crystals, where the driving agent is the mean curvature of the facets. Aikou reviews the essential role played by Finsler metrics in complex differential geometry. Chandler and Wong explain why parametrized jet bundles admit only Finsler metrics and develop machinery which they use to prove the Kobayashi conjecture (1960) and a special case of the Green–Griffiths (1979) conjecture. Bao and Robles focus on the flag and Ricci curvatures of Finsler manifolds, with an emphasis on Einstein metrics of Randers type. Rademacher gives a detailed and new account of his Sphere Theorem for nonreversible Finsler metrics. Shen's article explains why Finsler manifolds are colorful objects and examines the interplay among the flag, $S$-, and Landsberg curvatures in Finsler geometry.

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# A Sampler of Riemann–Finsler Geometry

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